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A hybrid GA–PS–SQP method to solve power system valve-point economic dispatch problems

J.S. Alsumait [a,*], J.K. Sykulski [a], A.K. Al-Othman [b]

[a] University of Southampton, Electronics and Computer Science School, Electrical Power Engineering Group, Highfield Southampton SO17 1BJ, United Kingdom
[b] The Public Authority for Applied Education and Training, College of Technical Studies, Electrical Engineering Department, Alrawda, 73452, P.O. Box 33198, Kuwait

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ABSTRACT

This study presents a new approach based on a hybrid algorithm consisting of Genetic Algorithm (GA), Pattern Search (PS) and Sequential Quadratic Programming (SQP) techniques to solve the well-known power system Economic dispatch problem (ED). GA is the main optimizer of the algorithm, whereas PS and SQP are used to fine tune the results of GA to increase confidence in the solution. For illustrative purposes, the algorithm has been applied to various test systems to assess its effectiveness. Furthermore, convergence characteristics and robustness of the proposed method have been explored through comparison with results reported in literature. The outcome is very encouraging and suggests that the hybrid GA–PS–SQP algorithm is very efficient in solving power system economic dispatch problem.

1. Introduction

Economic dispatch (ED) is an optimization problem where optimal generation for each generator is determined to minimise total fuel costs, subject to equality constraints on power balance and inequality constraints on power outputs. Moreover, transfer losses, generation rate changes and line flows may also be considered. The complexity of the problem increases dramatically with the number of units because of their combinatorial nature. For the sake of simplicity, the cost function for generating units is often approximated by a quadratic function.

A variety of techniques may be used to solve ED problems; some are based on classical optimization methods, such as linear or quadratic programming [1,2], while others use artificial intelligence or heuristic algorithms. Classical techniques are highly sensitive to a selection of the staring point and often converge to a local optimum or even diverge altogether. Linear programming methods are generally fast and reliable but use piecewise linear cost approximation which reduces accuracy. Non-linear programming methods, on the other hand, have convergence problems and often result in very complex algorithms. Newton based algorithms suffer from difficulties associated with handling a large number of inequality constraints [3]. More recently, heuristic search techniques – such as particle swarm optimization (PSO) [4–6] and genetic algorithm (GA) [7] – have also been considered in the context of ED. In addition, differential evolution algorithms were implemented to solve the ED problem [8–10]. Differential evolution (DE) is a stochastic search based method, which can present a simple structure, convergence speed, versatility, and robustness. However, DE fast convergence might lead the direction of the search toward a local optimal and premature solution. Finally, the use of harmony search (HS) method to find the global or near global solution for the ED problem can be found in [11,12]. HS is considered as a stochastic random search method, which does not need any information about the derivative. Nevertheless, HS has some insufficiencies associated with the premature convergence in its performance.

In the pursuit of the optimal solution for ED, various hybrid methods have been investigated and implemented [13–15]. A combined Particle Swarm Optimization and Sequential Quadratic Programming (PSO–SQP) algorithm was developed in [13], where PSO is the main optimizer and the SQP is used to fine tune the PSO solution. However, since SQP is a gradient dependent method, its application to non-continuous, non-differentiable and multimodal problems, such as ED, might not lead to an optimal solution. An increasing international concern about environment also affects the field of power generation where environmental issues are addressed directly. In another hybrid approach [14], the Differential

* Corresponding author.
E-mail addresses: jas2@soton.ac.uk (J.S. Alsumait), jks@soton.ac.uk (J.K. Sykulski), alothman@paaet.edu.kw (A.K. Al-Othman).

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Evolution (DE) and the Sequential Quadratic Programming (SQP) were combined into a single algorithm and used on 13 and 40 thermal units whose incremental fuel cost functions contain the valve-point loading effect. In [15] the authors combined three evolutionary methods to solve a fuzzy modelled Unit Commitment Problem (UCP). The three methods are Tabu Search (TS), Particle Swarm Optimization (PSO) and Sequential Quadratic Programming (SQP) (referred to as a hybrid TS–PSO–SQP). TS is used to solve the combinatorial sub-problem of the UCP. Then the non-linear programming sub-problem of the UCP is solved using the hybrid PSO–QSP technique.

Finally, a particular family of global optimization methods, known as Direct Search methods, originally introduced and developed by researchers in 1960s [16], has recently received some attention. The Direct Search methods are simply structured to explore a set of points, in the vicinity of the current position, looking for a smaller objective function value than the current one. This family includes Pattern Search (PS) algorithms, Simplex Methods (SM) (different from the simplex used in linear programming), Powell Optimization (PO) and others [17]. Direct Search methods, in contrast to more standard optimization methods, are often called derivative-free as they do not require any information about the gradient (or higher derivative) of the objective function when searching for an optimal solution. Therefore Direct Search methods are particularly appropriate for solving non-continuous, non-differentiable and multimodal (i.e. multiple local optima) optimization problems, such as the economic dispatch.

The main objective of this study is to introduce a hybrid method that combines the Genetic Algorithm (GA), Pattern Search (PS) and Sequential Quadratic Programming (SQP) – referred to as the hybrid GA–PS–SQP method – in the context of power system economic dispatch problem with a valve-point effect. The valve-point effect is a 'ripple' added to the generating unit's curve when each steam admission valve in a turbine starts to open. Therefore, to improve accuracy when using this model, an additional term representing the valve-point effect is added to the cost function as suggested in [18]. The addition of the valve-point effect poses a more challenging task to the proposed method since it increases the non-linearity of the search space as well as the number of local minima (see Fig. 1). The introduction of PS in the proposed hybrid algorithm as a refining search method has added additional factor of confidence in the final solution. In addition, the proposed method consists of a combination of three search methods that have been used for the first time together in the literature.

The proposed hybrid method has eliminated the need to provide a suitable starting point for PS and/or SQP. This feature led to the reduction of total execution time of the algorithm when compared to other reported methods. In a previous paper of the authors [19], finding a proper initial point for PS required an extended computational time to locate its best solution. The additional time came from the need to execute the algorithm with 100 different starting points to get the minimum fuel cost reported in [19]. On the other hand, the proposed algorithm needed only one single run (Case I only) to produce acceptable results that can be compared to the outcomes of the other methods. More details will be presented in the numerical result section.

The paper is organized as follows: Section 2 introduces the problem formulation; Section 3 presents a description of the proposed PS algorithm; the analysis and test results are included in Section 4, followed by concluding remarks.

2. Problem formulation

The traditional formulation of the economic dispatch problem is a minimization of summation of the fuel costs of the individual dispatchable generators subject to the real power balanced with the total load demand as well as the limits on generators outputs. In mathematical terms the problem may be stated as:

$$F = \sum_{i=1}^{N} F_i(P_i) \tag{1}$$

The incremental fuel cost functions of the generation units with valve-point loading are represented as follows [20]:

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + |e_i \times \sin(f_i \times (P_{i\,\min} - P_i))| \tag{2}$$

subject to

$$\sum_{i=1}^{N} P_{gi} = P_D + P_L \tag{3}$$

$$P_{gi_{\min}} < P_{gi} < P_{gi_{\max}}, i \in N_s \tag{4}$$

where F is the system overall cost function, N the number of generators in the system, d_i, b_i, c_i the constants of fuel function of generator number i, e_i, f_i the constants of the valve-point effect of generator number i, P_{gi} the active power generation of generator number i, P_D the total power system demand, P_L the total system transmission losses, $P_{gi_{\min}}$ the minimum limit on active power gen-

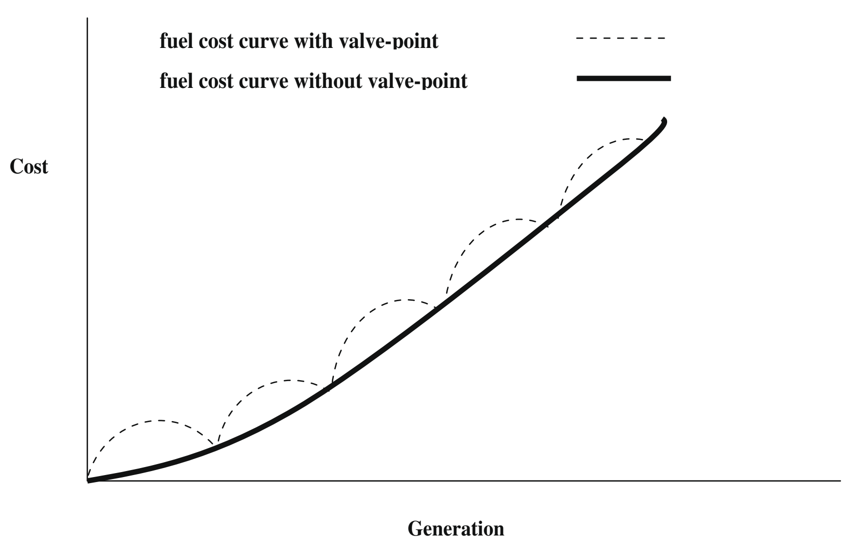

Fig. 1. The valve-point effect.

eration of generator i, $P_{gi_{max}}$ the maximum limit on active power generation of generator i, and N_s is the set of generators in the system.

The sinusoidal term added to the fuel cost function which models the valve-point effect creates ripples in the heat-rate curve and therefore introduces more local minima to the search space.

For the sake of simplicity, the system losses are ignored for all the test arrangements reported in this study.

3. Methods description

Due to the limitations on the length of this paper, and good coverage of the GA technique in the literature, only the PS and SQP method will be briefly mentioned here. A more comprehensive description of the technique may be found in the recent publication by the authors [17].

3.1. Pattern search

The Pattern Search (PS) is an evolutionary routine suitable for solving a variety of optimization problems that lie outside the scope of typical optimization tasks, and has the advantage of being very simple in concept, easy to implement and computationally efficient. Unlike, say, genetic algorithms [21,22], the PS has a flexible and well-balanced operator able to enhance and adapt the global as well as fine tune the local search. A helpful review of direct search methods for unconstrained optimization may be found in [17].

The PS algorithm computes a sequence of points that may or may not approach the optimum. First, a set of points called a *mesh* is established around an existing point (initial or from the previous step). If a point in the mesh is found to improve the objective function, the new point becomes the current point at the next iteration. The PS begins with an initial point X_0 supplied by the user. At the first iteration, with a scalar = 1 (called the *mesh size*), the pattern vectors (or *direction vectors*) are constructed as $[0\,1]$, $[1\,0]$, $[-1\,0]$ and $[0\ -1]\ +\ X_0$, and new mesh points are added as illustrated in Fig. 2. Objective functions are computed until a value smaller than for X_0 is found. If there is such a point, the poll is successful and the algorithm sets this point as equal to X_1 .

After a successful poll, the algorithm multiplies the current mesh size by 2, (called the expansion factor) and proceeds to iteration 2 with the following new points: $2*[1\,0]+X_1$,$2*[0\,1]+X_1$, $2*[-1\,0]+X_1$ and $2*[0\ -1]+X_1$. If a value smaller than for X_1 is found, X_2 is defined, the mesh size is increased by two and iterations continue. If at any stage the poll is unsuccessful, i.e. no point has an objective function smaller than the most recent value, the current point is not changed and the mesh size is reduced (e.g. by multiplying by 0.5, a contraction factor). These steps are repeated until the optimum is found, that is one of the terminating conditions occurs, for example the mesh size is less than the set tolerance, the maximum number of iterations has been reached, the change in the value of the objective function is very small, or similar. The algorithm is depicted in Fig. 3.

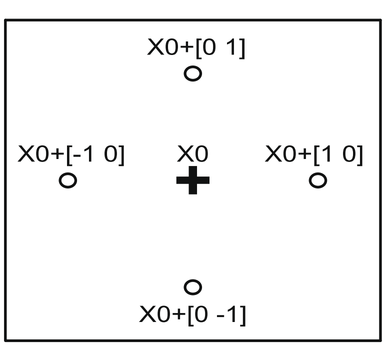

Fig. 2. PS mesh points and the pattern.

3.2. Sequential quadratic Programming

The solution of the Kuhn–Tucker (KT) equations forms the basis of many non-linear programming algorithms. These algorithms attempt to compute the Lagrange multipliers directly. Constrained quasi-Newton methods guarantee super linear convergence by accumulating second order information regarding the KT equations using a quasi-Newton updating procedure [23]. These methods are commonly referred to as Sequential Quadratic Programming (SQP) methods, since a QP sub-problem is solved at each major iteration (also known as Iterative Quadratic Programming, Recursive Quadratic Programming, and Constrained Variable Metric methods). The QP optimization problem can be described as follows:

$$\min\left(\frac{1}{2}d_k^T H_k d_k + \nabla f\ x_k^T\ d_k\right)$$
$$\text{subject to}\quad [\nabla g x_k]^T d_k + g_i x_k = 0 \quad i = 1,\ldots,m_e$$
$$[\nabla g x_k]^T d_k + g_i x_k \le 0 \quad i = m_e+1,\ldots,m$$

where H_k is the Hessian matrix of the Lagrange function$L(x,\lambda) = f(x) + \sum_{i=1}^{m}\lambda_i * g_i(x)d_k$ is a basis of the search direction of the k^{th} iteration.

SQP can be decomposed into three main stages:

- Updating of the Hessian matrix of the Lagrangian function.
- Line search and merit function calculation.
- Quadratic programming problem solution.

A convergence test is made at each iteration, after the solution of the quadratic programming problem until the control variables, gradient of functions and objective function reaches a specified tolerance value [24,25]. It should be mentioned that SQP method needs a starting point, which will be provided from the second phase of computation (PS phase).

Many ideas have been suggested to ensure that the solution satisfies the constraint [26]. For example, the constraint can be augmented with the objective function using Lagrange multipliers. In this way the size of the problem will increase by introducing new parameters. In this study, the Pattern Search (PS) method handles constraints by using augmented Lagrangian to solve the non-linear constrained economic dispatch problem [27–30]. The variables' bounds and linear constraints are handled separately from non-linear constraints. Thus a sub-problem is formulated and solved, having the objective function and non-linear constraint function, using the Lagrangian and the penalty factors. Such a sub-problem is minimized using a pattern search method, where the linear constraints and bounds are satisfied. For more explanation on how PS handles constraints refer to [29,31,32].

4. Numerical results

In order to asses the effectiveness and robustness of the proposed hybrid method, several test cases of economic dispatch with valve-point effect have been considered. For simplicity, transmission losses are ignored in all test cases (PL in Eq. (3) is set to zero). The non-linear minimization problem formulation of all test cases has been solved using the predefined functions ga, pattern search and fmincon incorporated into the GA & DS toolbox of Matlab [31]. Consequently, cost coefficients of the fuel cost and the combined objective function for the considered test cases were coded in Matlab environment.

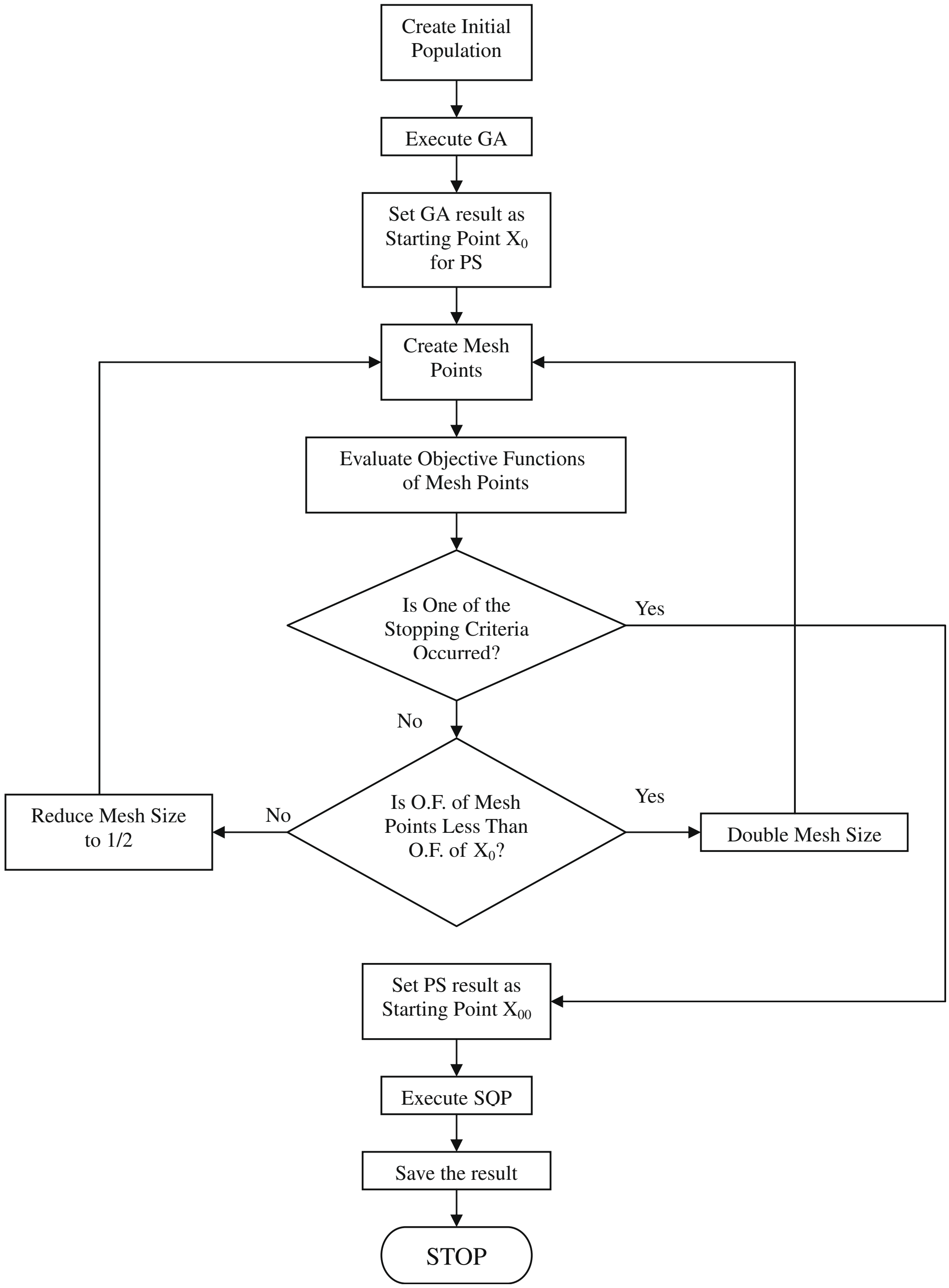

Fig. 3. Flow chart of the proposed hybrid method.

Initially, several runs were carried out with different values of the key parameters of GA, PS and SQP, such as migration rate, cross-over rate, population size, initial mesh size, and mesh expansion and contraction factors. For the GA, different values for migration rates and cross-over were chosen for each case, whereas the population size for GA was set to 100. The mesh size and the mesh expansion and contraction factors were selected as 1, 2, and 0.5 respectively. As for the terminating criteria in PS iterations, all tolerances were set to 10–6 and the maximum number of iterations and function evaluations were chosen to be 1000. All runs were conducted using a modest laptop computer with 1 GHz Pentium 3 processor and 256 MB of RAM, so the comparisons of computing times with those given in literature should be fair.

4.1. Case I: three generating units

Three generating units have been modelled using a quadratic cost function and with the effects of the valve-point loading included. All data (upper and lower bounds for the units and fuel cost coefficients *a*, *b*, *c*, *e*, and *f*) are given in [20,32], and the load demand is 850 MW.

The hybrid GA–PS–SQP algorithm has been executed 100 times to study its performance and effectiveness. The proposed algorithm has produced the same final result for the 100 runs. It was concluded that Case I needs only single run to reach its optimal or near optimal solution. The execution times have been compared with other evolutionary methods, such as Genetic Algorithm (GA), Evolutionary Programming (EP) and Particle Swarm Optimization

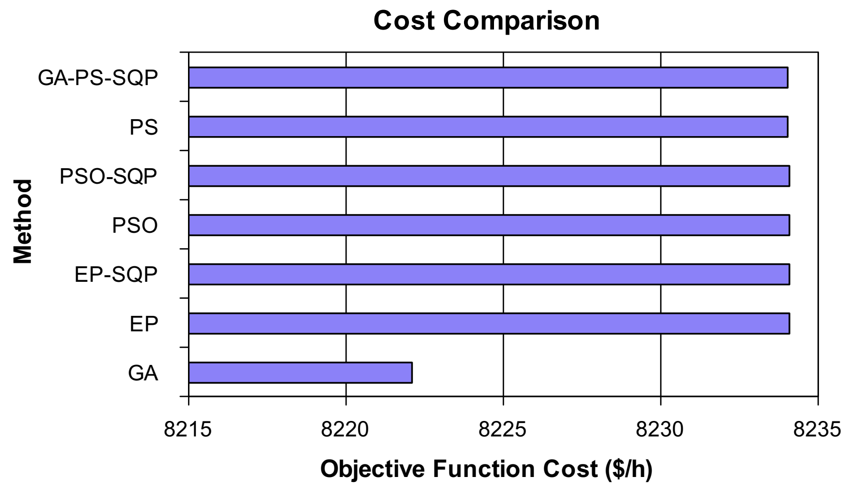

Fig. 4. Minimum cost comparison.

Table 1
Generator loading and total fuel cost with the total load demand of 850 MW (Case I).

Method	P_1 (MW)	P_2 (MW)	P_3 (MW)	P_D (MW)
GA	398.7	399.6	50.1	848.4
EP	300.3	400.0	149.7	850.0
EP–SQP	300.3	400.0	149.7	850.0
PSO	300.3	400.0	149.7	850.0
PSO–SQP	300.3	400.0	149.7	850.0
PS	300.3	399.9	149.7	850.0
GA–PS–SQP	300.3	400.0	149.7	850.0
Total fuel cost ($/h) = 8234.1				

Table 2
Comparison of execution times and costs.

Method	Mean time (s)	Best cost ($/h)	Mean cost ($/h)
GA	35.80	8222.1	8234.7
EP	6.78	8234.1	8234.2
EP–SQP	5.12	8234.1	8234.1
PSO	4.37	8234.1	8234.7
PSO–SQP	3.37	8234.1	8234.1
PS	0.81	8234.1	8352.4
GA–PS–SQP	15.28	8234.1	8234.1

Table 3
Generator loading and fuel cost determined by the GA–PS–SQP hybrid method with total load demand of 1800 MW.

Generator	Unit generation (MW)
Pg_1	628.31
Pg_2	148.50
Pg_3	224.03
Pg_4	109.75
Pg_5	109.85
Pg_6	60.00
Pg_7	109.86
Pg_8	109.83
Pg_9	109.86
Pg_{10}	40.00
Pg_{11}	40.00
Pg_{12}	55.00
Pg_{13}	55.00
Total cost ($/h)	17964.25

(PSO), presented in [13]. Moreover, previous results from the implementation of the Patter Search (PS) method in ED problems have also been added [19]. This numerical experiment compares the performance of the proposed hybrid algorithm with the other methods in terms of the dispatching cost and the speed of convergence. Table 1 shows the optimal solutions determined by the different methods, whereas the execution times and cost comparison are shown in Table 2.

All methods (except GA) give an almost identical 'best' solution, whereas 'mean' costs differ slightly. The mean execution time for the proposed hybrid method is worse than for the other methods, except for GA, due to three consecutive searches being applied when seeking the best solution. However, the proposed algorithm requires only one run (in this case only) to achieve its final solution, whereas the other methods needed 100 runs. The reported times in other methods were the mean execution times, and the total run time in this case should be the mean multiplied by the number of runs (100). For a fair comparison in computation time, the total run times of the other algorithms should be compared with the execution time of the proposed method. For example, the total computational time for PS (100 runs) was 22.14 s, and when compared to GA–PS–SQP's 15.28 s, one can find that the proposed algorithm has saved about 30% of computational time. Figs. 4 and 5 compare the results of the methods in terms of the minimum cost and the best execution time, respectively.

For GA, the population size, migration rate and cross-over rate were set to 100, 0.76 and 0.4, respectively, while the parameters for the PS were stated previously.

4.2. Case II: 13 generating units

In this test there are 13-generating units, while quadratic cost functions combined with the effects of valve-point loading have

Table 4
Comparison of GA–PS–QSP results (Case II).

Evolution method	Mean time (s)	Minimum cost ($/h)	Mean cost ($/h)
EP	157.43	17,994	18,127
EP–SQP	121.93	17,991	18,107
PSO	77.37	18,031	18,206
PSO–SQP	33.97	17,970	18,030
PS	5.88	17,969	18,089
HQPSO(5)	–	17,964	18,274
FAPSO–NM	6.8	17,964	17,964
IHS	–	17,960	17,965
DE	1050.8	17,964	17,965
GA–PS–QSP	11.06	17,964	18,199

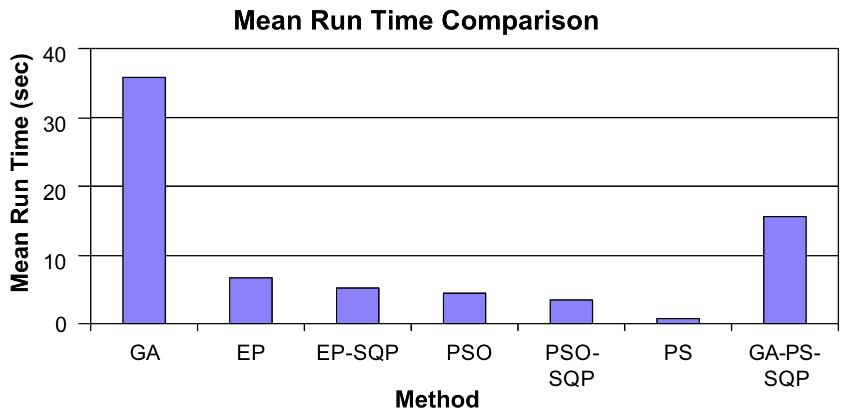

Fig. 5. Mean execution time comparison (with GA not meeting all constraints).

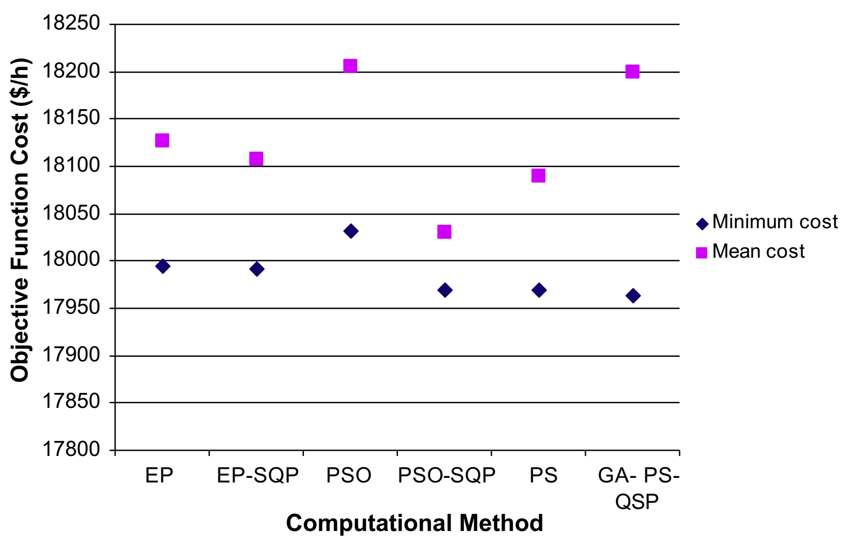

Fig. 6. Minimum cost comparison (Case II).

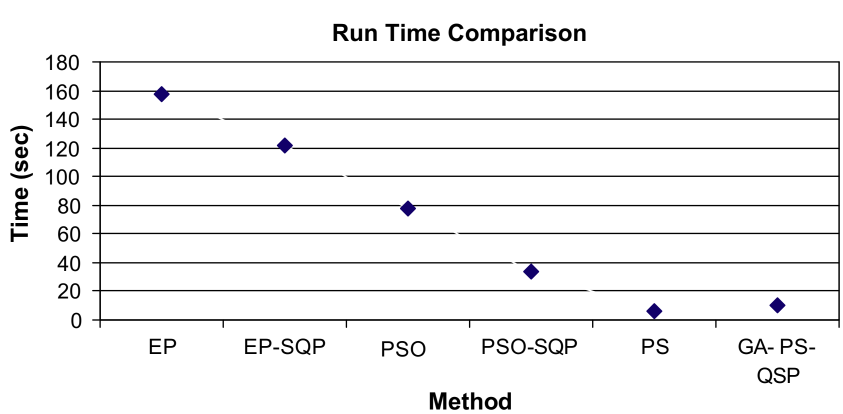

Fig. 7. Best execution time comparison (Case II).

been used as before. All data for the 13 generators may be found in [32,33] and the load demand is 1800 MW. The GA–PS–SQP algorithm has been executed 100 times. Similar comparisons as for Case I are summarized by Tables 3 and 4. The results for the other methods are taken from [4,5,9,12,13,19].

In this case the GA–PS–QSP hybrid method outperforms all other algorithms in terms of achieving the best minimum cost (although differences are quite small), while at the same time offering significant saving in computing times – except for the PS method (see also Figs. 6 and 7). It appears that the proposed

Table 5
Generator loadings and fuel costs determined by GA–PS–SQP (Case III).

Generator	Generator production (MW)	Generator	Generator production (MW)	Generator	Generator production (MW)	Generator	Generator production (MW)
Pg_1	110.97	Pg_{11}	168.80	Pg_{21}	523.28	Pg_{31}	190.00
Pg_2	111.02	Pg_{12}	168.80	Pg_{22}	523.28	Pg_{32}	190.00
Pg_3	120.00	Pg_{13}	214.76	Pg_{23}	523.28	Pg_{33}	190.00
Pg_4	179.73	Pg_{14}	394.28	Pg_{24}	523.28	Pg_{34}	164.80
Pg_5	88.27	Pg_{15}	304.52	Pg_{25}	523.28	Pg_{35}	200.00
Pg_6	140.00	Pg_{16}	304.52	Pg_{26}	523.28	Pg_{36}	200.00
Pg_7	259.60	Pg_{17}	489.28	Pg_{27}	10.00	Pg_{37}	110.00
Pg_8	284.60	Pg_{18}	489.28	Pg_{28}	10.00	Pg_{38}	110.00
Pg_9	284.60	Pg_{19}	511.28	Pg_{29}	10.00	Pg_{39}	110.00
Pg_{10}	130.00	Pg_{20}	511.28	Pg_{30}	88.66	Pg_{40}	511.28
ΣPg_i = 10,500 MW				Total cost: $121458.14			

Table 6
Comparison of GA–PS–SQP results (Case III).

Method	Mean time (S)	Minimum cost ($)	Mean cost ($)
EP	1167.35	122,624	123,382
EP–SQP	997.73	122,324	122,379
PSO	933.39	123,930	124,154
PSO–SQP	733.97	122,094	122,245
PS	42.98	121,415	122,333
FAPSO–NM	40	121,420	121,419
DE	72.94	121,416	121,423
GA–PS–SQP	46.98	121,458	122,039

algorithm performs better as the problem becomes larger and more complex. The migration and cross-over rates for GA have been changed in this case to 0.64 and 0.3, respectively, whereas the population size is the same as in the previous case. For the record, the best solution time and the minimum time for the 100 runs were 11.06 s and 6.77 s, respectively.

The proposed hybrid method has generated very satisfactory solutions, all 100 being within 2.3% of the best result. The maximum cost and the total execution time for the 100 runs were 18,392 $/h and 1054.9 s, respectively.

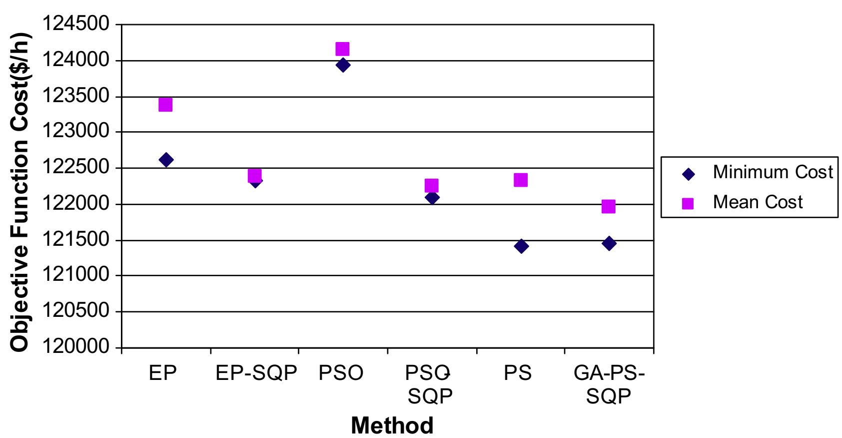

Fig. 8. Cost Comparison for Case III.

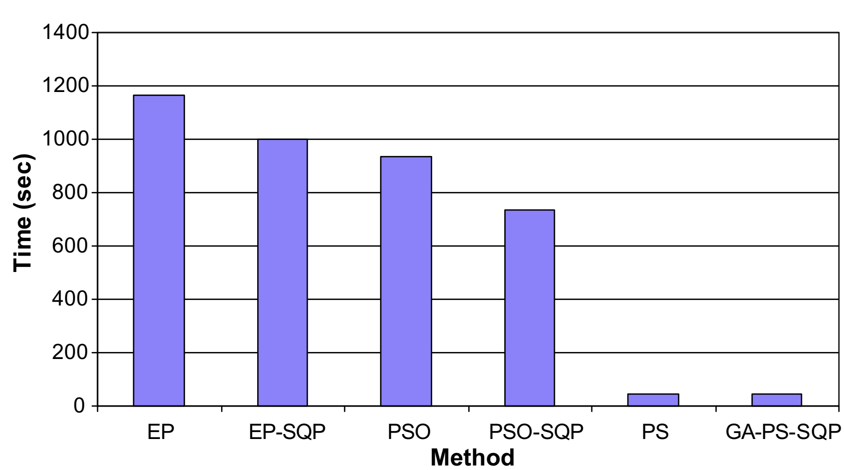

Fig. 9. Best times comparison (Case III).

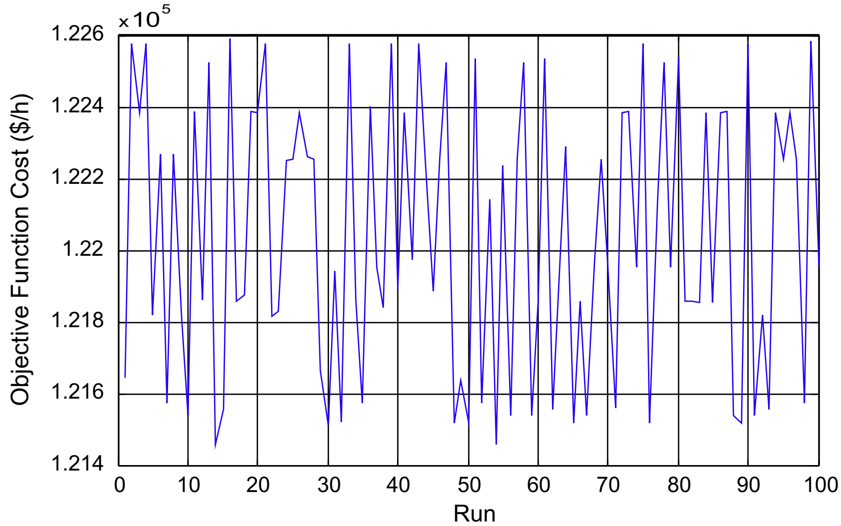

Fig. 10. Objective function values for 100 different starting points (Case III).

4.3. Case III: 40 generating units

The final test case consists of 40 generating units with full data given in [32,34]. The GA–PS–SQP algorithm has been executed a hundred times and the results and comparison with other methods from [5,9,13,19] are given in Tables 5 and 6, respectively, while Figs. 8 and 9 show the comparison of costs and best times for all methods. The load demand is 10,500 MW.

Fig. 10 illustrates the quality of the optimum depending on the starting point provided by the hybrid GA–PS to the SQP algorithm. The tendencies and the properties of the algorithm are similar to those observed when studying Case II. Overall, the proposed hybrid method yields the best mean cost of all the methods compared, at significant savings of computational effort. These short computing times allow for more cases to be studied with the aim of increasing the confidence in the final solution. In addition, all results from the 100 runs are within 1% of the best value. It may therefore be concluded that the first stage (i.e. the outcome of the PS) provides a good staring point to the final search method to ensure that all results are global or near global solutions. In this case the migration rate, cross-over rate and population size are the same as for Case II, and the total computation time for the 100 runs is 4467.64 s.

One of the identified advantages of combining the three techniques into a hybrid GA–PS–SQP is to do with the removal of the requirement to provide an initial (staring) point for the algorithm to commence the search. The PS technique on its own, successfully implemented and reported in the previous paper [19], relies on a good initial 'guess' making the technique more susceptible to getting trapped in local minima. In the proposed hybrid method, the initial search based on the use of the GA does not require the user to provide such a starting value as the search is performed automatically. The tests undertaken have confirmed that this indeed makes the whole optimization process more robust and explains why the error bound of all solutions is now so narrow, much better than when using the other techniques.

5. Conclusions

This paper describes a novel hybrid approach based on a combination of Genetic Algorithm (GA), Pattern Search (PS) and Sequential Quadratic Programming optimization (SQP) to study power system economic dispatch problems, taking account of the valve-point effect. Three test cases (numerical experiments) have been studied, consisting of 3, 13 and 40 generators, respectively, and comparisons of the quality of the solution and performance have been conducted against Evolutionary Programming (EP), Particle Swarm Optimization (PSO), hybrid EP–SQP (Sequential Quadratic Programming) and hybrid PSO–SQP methods. The results demonstrate that the proposed scheme outperforms the other methods in terms of better optimal solutions and significant reduction of computing times. The economy of computation is particularly noticeable for more complicated problems with larger number of units. Furthermore, the GA–PS–SQP technique has overcome an important drawback of the PS or SQP methods that is the need to supply a suitable starting point. This shortcoming of the PS and SQP methods was highlighted in the previous work of the authors as it makes any optimization method relying on a good choice of the initial point possibly more susceptible to getting trapped in local minima, although the much improved speed of computation allows for additional searches to be made to increase the confidence in the solution. The hybrid GA–PS–SQP algorithm, on the other hand, does not require the user to specify the starting point as it is generated automatically for the final SQP stage by the initial GA–PS phase. Moreover, the performance of the proposed hybrid method improves with the increase of size and complexity of the system. Overall, the proposed algorithm has been shown to perform extremely well for solving economic dispatch problems.

References

[1] Adler RB, Fischl R. Security constrained economic dispatch with participation factors based on worst case bus load variations. Power Apparatus Syst IEEE Trans 1977;96:347–56.
[2] Bui RT, Ghaderpanah S. Real power rescheduling and security assessment. IEEE Trans Power Apparatus Syst 1982;PAS-101:2906–15.
[3] Pancholi RK, Swarup KS. Particle swarm optimization for security constrained economic dispatch. In: International conference on intelligent sensing and information processing, (IEEE Cat. No. 04EX783), Chennai, India; 2004. p. 7–12.
[4] Dos Santos Coelho L, Mariani VC. Particle swarm approach based on quantum mechanics and harmonic oscillator potential well for economic load dispatch with valve-point effects. Energy Convers Manage 2008;49:3080–5.
[5] Niknam T. A new fuzzy adaptive hybrid particle swarm optimization algorithm for non-linear, non-smooth and non-convex economic dispatch problem. Appl Energy 2010;87(1):327–39.
[6] Cai J, Ma X, Li L, Haipeng P. Chaotic particle swarm optimization for economic dispatch considering the generator constraints. Energy Convers Manage 2007;48:645–53.
[7] Youssef HK, El-Naggar KM. Genetic based algorithm for security constrained power system economic dispatch. Electr Power Syst Res 2000;53:47–51.
[8] Coelho LDS, Mariani VC. Improved differential evolution algorithms for handling economic dispatch optimization with generator constraints. Energy Convers Manage 2007;48:1631–9.
[9] Noman N, Iba H. Differential evolution for economic load dispatch problems. Electr Power Syst Res 2008;78:1322–31.

[10] Coelho LDS, Souza RCT, Mariani VC. Improved differential evolution approach based on cultural algorithm and diversity measure applied to solve economic load dispatch problems. Math Comput Simulat 2009;79:3136–47.
[11] Fesanghary M, Ardehali MM. A novel meta-heuristic optimization methodology for solving various types of economic dispatch problem. Energy 2009;34:757–66.
[12] Coelho LDS, Mariani VC. An improved harmony search algorithm for power economic load dispatch. Energy Convers Manage 2009;50:2522–6.
[13] Victoire TAA, Jeyakumar AE. Hybrid PSO-SQP for economic dispatch with valve-point effect. Electr Power Syst Res 2004;71:51–9.
[14] Coelho LS, Mariani VC. Combining of chaotic differential evolution and quadratic programming for economic dispatch optimization with valve-point effect. Power Syst IEEE Trans 2006;21:989–96.
[15] Victoire TAA, Jeyakumar AE. A tabu search based hybrid optimization approach for a fuzzy modelled unit commitment problem. Electr Power Syst Res 2006;76:413–25.
[16] Hooke R, Jeeves TA. Direct search solution of numerical and statistical problems. J Assoc Comput Machin 1961;8:212–29.
[17] Lewis RM, Torczon V, Trosset MW. Direct search methods: then and now. J Comput Appl Math 2000;124:191–207.
[18] Kim JO, Shin DJ, Park JN, Singh C. Atavistic genetic algorithm for economic dispatch with valve point effect. Electr Power Syst Res 2002;62:201–7.
[19] Al-Sumait JS, Al-Othman AK, Sykulski JK. Application of pattern search method to power system valve-point economic load dispatch. Int J Electr Power Energy Syst 2007;29:720–30.
[20] Walters DC, Sheble GB. Genetic algorithm solution of economic dispatch with valve point loading. Power Syst IEEE Trans 1993;8:1325–32.
[21] Goldberg DE. Genetic algorithms in search, optimization, and machine learning. Harlow: Addison-Wesley; 1989.
[22] Michalewicz Z. Genetic algorithms + data structures = evolution programs, 3rd rev. and extended ed.. Berlin, New York: Springer-Verlag; 1996.
[23] Powell MJD. A note on quasi-Newton formulae for sparse second derivative matrices. Math Program 1981;20:144–51.
[24] Powell MJD. A fast algorithm for nonlinearly constrained optimization calculations numerical analysis. In: Watson GA, editor. Lecture notes in mathematics. Springer-Verlag; 1978. vol. 630.
[25] Powell MJD. Convergence properties of algorithms for nonlinear optimization. SIAM Rev 1986;28:487–500.
[26] Lewis RM, Torczon V. Pattern search algorithms for linearly constrained minimization. SIAM J Optim 1999;10:917–41.
[27] Lewis RM, Torczon V. Pattern search algorithms for bound constrained minimization. SIAM J Optim 1999;9:1082–99.
[28] Lewis RM, Torczon V. A globally convergent augmented Lagrangian pattern search algorithm for optimization with general constraints and simple bounds. SIAM J Optim 2001;12:1075–89.
[29] Torczon V. On the convergence of pattern search algorithms. SIAM J Optim 1997;7:1–25.
[30] Conn AR, Gould NIM, Toint PL. A Globally convergent augmented Lagrangian algorithm for optimization with general constraints and simple bounds. SIAM J Numer Anal 1991;28:545–72.
[31] Genetic Algorithm and Direct Search Toolbox for use with Matlab user's guide, second ed., version 7.6.0.324, release R2008a. The Math works Inc.
[32] Sinha N, Chakrabarti R, Chattopadhyay PK. Evolutionary programming techniques for economic load dispatch. Evolut Comput IEEE Trans 2003;7:83–94.
[33] Wong KP, Wong YW. Thermal generator scheduling using hybrid genetic simulated-annealing. Generation, transmission and distribution. IEE Proc 1995;142:372–80.
[34] Chen PH, Chang HC. Large-scale economic dispatch by genetic algorithm. Power Syst IEEE Trans 1995;10:1919–26.